Variational Data Assimilation for Short Range Atmospheric Dispersion of Radionuclides Based on Measurements Provided by an Unmanned Aerial Vehicle

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Scope

The ability to detect and monitor radiation releases or nuclear threats is important to public health protection and homeland security. Unmanned aerial vehicles (UAV), more often referred to as drones, are quickly becoming commonplace. Airborne radiological three-dimensional (3D) data can enhance the prediction accuracy of dispersion models used in tracking atmospheric release of radioactive pollution. The airborne mobile sensors can complement the stationary networks and improve the real-time dose prediction. Development and testing of the data assimilation methodology is the objective of the current ongoing work.

A sensor package mounted on an UAV has been developed for monitoring the radiation levels in plumes from the isotope production and reactor process stacks in support of environmental monitoring on the CNL site. The application of variational data assimilation based on a simple Gaussian plume model for radionuclides is considered with the data collected.

UAV Radiological 3D Data

CNL has developed a light and compact radiation sensor package that is carried onboard of an UAV. The current sensor package measures gamma radiation levels and spectra. It is a self-contained unit weighing three-quarters of a kilogram that includes a caesium iodide-based detector (Hamamatsu C12137-01), a microprocessor with data storage, position sensors (both GPS and altitude), wireless transceiver, power supply, and a housing and mount to the UAV. Custom software allows control of the sensor package and has a live display and near-real-time 3D mapping of the radiation data (less than 2 seconds lag).

Figure 1: UAV monitoring system developed at CNL.

The radio transmitter-receiver system communicates the measurements to the ground station.

Figure 2: Portable ground station that communicates with the UAV.

The gamma radiation detector has an energy range of 30-2000 keV with an energy resolution of 8.5 keV. The range of dose measurements is 0.001-10 μSv/h with a measurement error of 20%. GPS coordinates of this airborne sensor are monitored with an accuracy of 5-10 m.

UAV Dose-Rate Measurements

Measurements are typically performed with the UAV in the vicinity of the stack of the National Research Universal (NRU) reactor at CNL. Routine gamma-emissions from the 50m-high NRU stack are typically dominated by Ar-41 released at the rate of 2.1E+14 Bq/week.

Three to four flights are usually performed per day and each flight lasts about 10-15 minutes. For instance, 3D data for the radioactive plume was collected at 1Hz frequency on a clear day (October 3, 2017) between 5 and 7 pm right before and during the sunset, i.e., during the late afternoon transition of atmospheric boundary layer. Apart from the UAV, emissions are also recorded by a CNL network of monitoring stations. During flights meteorological data at 0.1 Hz is collected from 30m and 60m about ground levels at the tower at Perch Lake located approximately 1,800m upwind from the stack.

Figure 3: Illustration of UAV flight to collect radiation dose rates around the NRU stack.

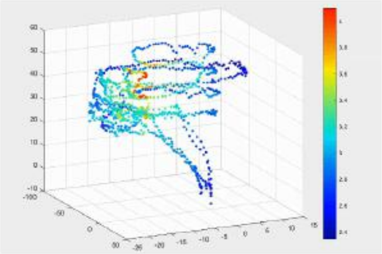

Figure 4: Depiction of radiation dose rates in the plume measured as the UAV flies near the NRU stack. The 3D UAV position is plotted every second and the dose rate value is shown at each point on a relative colour scale.

Computational Model

Gaussian Plume

The plume release from the stack is assumed to be Gaussian in shape so that the activity concentration at an arbitrary point in space, $\chi(x,y,z)$, is given by the equation:

$$\chi(x,y,z) = \frac{Q}{2\pi u \sigma_y(x)\sigma_z(x)} \exp\left(-\frac{y^2}{2\sigma_y^2}\right) \left[\exp\left(-\frac{(z-H)^2}{2\sigma_z^2}\right) + \exp\left(-\frac{(z+H)^2}{2\sigma_z^2}\right)\right], \quad (1)$$

where Q is the radionuclide release rate, u is the wind speed, and H is the source height. σ_y and σ_z are the Gaussian dispersion parameters in the cross-wind and vertical directions, respectively, given by:

$$\sigma_y = \frac{a_\sigma x}{(1+b_\sigma x)^{c_\sigma}}, \quad \sigma_z = \frac{d_\sigma x}{(1+e_\sigma x)^{f_\sigma}} \quad (2)$$

where the coefficients a_σ, b_σ, c_σ, d_σ, e_σ, and f_σ are determined by the atmospheric stability.

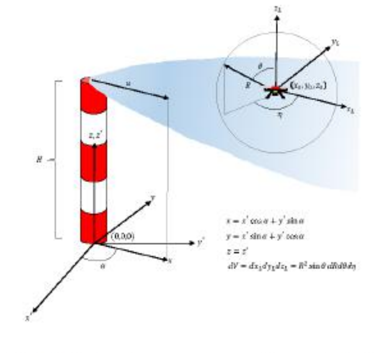

Figure 5: Illustration of the coordinate systems used in the computational model: (x', y', z') is the stack coordinate system, (x, y, z) is the plume coordinate system in which x-axis aligns with the wind direction α, (θ, η, R) is the spherical coordinate system at the drone measurement location (x_0, y_0, z_0). The dose rate integration is performed in the domain of the Gaussian plume.

Gamma Count Rate at Detector

The gamma count rate at the location of the detector is a function of the spatial distribution of contaminant in the plume as well as the interaction of gammas with air. The scalar flux of gammas, ϕ_γ, is determined through integration:

$$\phi_\gamma(x_0, y_0, z_0) = \int\int\int \frac{B(E_\gamma, \mu R)\exp(-\mu R)\chi(x,y,z)dV}{4\pi R^2}, \quad (3)$$

where the buildup $B(E_\gamma, \mu R)$ is a function of of the gamma energy, E_γ, as well as the linear attenuation coefficient in air, μ.

The properties of the CsI(Th) detector allow definition of a constant K that enables conversion between ϕ_γ and the count rate (relatable to dose rate) measured by the detector, i.e., $D(x_0, y_0, z_0) = K\phi_\gamma(x_0, y_0, z_0)$. For the given detector, it has been determined $K = 3.8558 \times 10^{-4} [m^2]$.

Data Assimilation

The computational model is parameterized with d parameters $\mathbf{p} = [p_1, \cdots, p_d]^T \in \Omega \subset \mathbb{R}^d$, where Ω is a bounded domain containing the set of admissible parameters. The observations (dose rate measurements), $\mathbf{y}_o$, are related to the system state by the equation

$$\mathbf{y}_o = \mathcal{H}(\mathbf{p}) + \epsilon^o, \quad (4)$$

where $\mathcal{H} = \phi_\gamma$ is a non-linear operator and ϵ^o is a vector of observational errors. The optimal input model parameters, $\hat{\mathbf{p}}$, that minimize the discrepancy between model outputs and observations:

$$\hat{\mathbf{p}} = \min_{\mathbf{p}\in\Omega} \mathcal{J}(\mathbf{p}), \quad (5)$$

where $\mathcal{J}(\mathbf{p}) : \Omega \to \mathbb{R}$ is an objective or cost function to be minimized.

Assuming $\mathbf{p}_b$ is the vector of background parameters, the objective function has the form

$$\mathcal{J}(\mathbf{p}) = \frac{1}{2}\sum_{k=1}^{P} \frac{(p_k - p_{b,k})^2}{\sigma_{b,k}^2} + \frac{1}{2}\sum_{i=1}^{N} \frac{(\mathcal{H}(\mathbf{p})_i - y_i)^2}{\sigma_i^2}, \quad (6)$$

where $\sigma_{b,k}$ and σ_i account for the uncertainties in background parameter values and measurements, respectively.

The nine physical parameters to be determined by solving Equation 5 are

$$\mathbf{p} = [Q, u, \alpha, a_\sigma, b_\sigma, c_\sigma, d_\sigma, e_\sigma, f_\sigma]^T. \quad (7)$$

These physical parameters are used to define the Gaussian plume described by Equation 1. The output of the assimilation procedure is thus the Gaussian plume that best describes the dose rate measurements made by the UAV.

Implementation and Ongoing Work

Solution of Equation 5 is being sought based on the Broyden-Fletcher-Goldfarb-Shanno algorithm (Limited-memory BFGS, or L-BFGS). Application of a BFGS method to this problem was previously considered by Quélo et al (2005). The data assimilation procedure will require multiple evaluations of the dose rate integral in Equation 3 to determine the gradient of $\mathcal{J}(\mathbf{p})$ with respect to parameters p. The solution framework is being written using a combination of C++ and Python functions, including functions from the SciPy library. The extensive optimization of the integration procedure in Equation 3 has been performed so as to make the solution procedure computationally feasible.

References

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